The mathematical equivalence of Keynesianism and monetarism

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Abstract

It has happened often in physics that a single phenomenon is explained, or a single puzzle resolved, by two theories that seem at first sight to be completely divergent but are later shown to be equivalent. Examples that spring to mind are Heisenberg's matrix mechanics and Schrodinger's wave mechanics or the quantum electrodynamic theories of Tomonaga, Schwinger and Feynman. In macroeconomics, the second half of the 20th century was dominated by the dispute between Keynesianism and monetarism, especially their divergent explanations of recessions, a dispute that continues to this day. This paper demonstrates that the conflict hinges on a simple dimensional misinterpretation of one of the variables in the quantity theory of money. At their heart, the two theories are equivalent.

Keywords Money; velocity of money; dimensional analysis; aggregate demand

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INTRODUCTION

The dimensions of the velocity of money are derived from the equation for the quantity theory of money.

$$M \cdot v = Y \tag{1}$$

where M is the quantity of money in an economy in dollars, v is its velocity and Y is the spending in a given time period. Since Y is usually taken to be the GDP and is measured in dollars per year, and M is measured in dollars, v, the residual, must be a number per year. Its dimensions are therefore t^{-1}.

The equation above has been the subject of considerable debate. But the debate is mainly about the stability of velocity and the causal relationship among the three variables in the equation. The equation itself is regarded as an accounting identity and therefore noncontroversial. The dimensional homogeneity of the equation is taken to be proof of its physical truth. It is this assumption that we will now investigate.

We begin by examining variables which have a dimension t^{-1} and which figure in real world physical relationships.

Case 1

The equation relating the frequency, wavelength and velocity of light is

$$\lambda \cdot \nu = c \qquad (2)$$

where λ is the wavelength in metres, ν is the frequency, and c is the speed of light in metres/second. The dimensions of frequency are thus $t\text{-}^{1}$, which are the same as the dimensions of the velocity of money.

If we conduct an experiment to test this relationship for red light over a period of 1 second, we find that light travels a distance of 3×10^8 metres. Since the wavelength of red light is 670×10^{-9} m, over this distance we observe that the waveform repeats itself 4.48×10^{14} times. The frequency of light is therefore 4.48×10^{14}/sec.

If instead we conduct the experiment over a period of $1/(3 \times 10^8)$ seconds, we find that light travels a distance of 1 m. Over this distance we would find that the the waveform repeats itself 1492537.31343 times. Elementary arithmetic shows that the frequency of light is therefore 4.48×10^{14}/sec. In other words, the frequency of light is independent of the period of time over which we measure it.

Case 2

A pipe carries water at a rate of 25 cm^3/sec.

We collect the water in a container over a period of 1 hour or 3,600 seconds and find that the volume of water collected is 90,000 cm^3. When the experiment is conducted over a period of 1 minute we find that the volume of water collected is 1500 cm^3.

In both cases we calculate and find that the flow rate is 25 cm^3/sec, which is of course what we would expect. As in Case 1, the result we get for the quantity whose dimensions are L^3t^{-1} is independent of the time period over which we choose to perform the measurement.

Case 3

A stone is dropped from a height of 500 m. We assume that the acceleration due to gravity is 10m/sec^2.

At the beginning of the fall the stone's velocity (dimensions: Lt^{-1}) is 0 m/sec. At the end of 1 sec it is 10 m/sec. At the end of 9 seconds the velocity is 90 m/sec. And at the end of 10 seconds the velocity is 100 m/sec.

In the first second the stone travels a distance of 5m. The average velocity in the first second is thus 5m/sec. During the entire 10 seconds the stone travels a distance of 500 m. The average velocity over the entire period is thus 50 m/sec. In the tenth second the stone travels a distance of 95 m. The average velocity in the 10th second is thus 95 m/sec.

In this situation, where the quantity in which time figures as a t^{-1} dimension is increasing constantly, the ratio of the quantity in the first second to the quantity in the entire time period is less than 1. The ratio of the quantity in the last second to the quantity in the entire time period is greater than 1. And the ratio of the quantity in one time period to the quantity in another time period (say, its value in the first second to that in the second second or its value in the second second to that in the third second) keeps varying.

Case 4

A stone thrown up from the ground at a speed of 100 m/sec is opposed by an acceleration of 10 m/sec^2. It follows an exactly opposite trajectory as the stone in the previous example, slowing down until at a height of 500 metres its velocity becomes 0 m/sec.

In this case, the quantity where time figures as a t^{-1} dimension is slowing at a constant rate. The ratio of the quantity in the first time period to the quantity in the entire time period is greater than 1. The ratio of the quantity in the last time period to the quantity in the entire time period is less than 1. And the ratio of the quantity in one time period to the quantity in another time period keeps varying.

These results can be summed up as follows. When the quantity in which time figures as a t^{-1} dimension is constant, its measured value is independent of the time period over which we make the measurement. When the quantity in which time figures as a t^{-1} dimension is constantly increasing or constantly decreasing, then the ratio of its values in any two sub-periods is not constant. The ratio of its value in the first sub-period to its value in the total period of measurement is not equal to one.

This is not a peculiarity of physical quantities in which time figures as t^{-1} but follows from the mathematics. Consider the density of a body in kg/m^3. If we divide the body into ten equal volumes (the quantity that occurs in the denominator in the definition of density), all of them will have equal density if the body is of constant density. If the density of one of those ten parts is higher than the density of the body as a whole, then the density of another part will have to be lower than the density of the body as a whole.

The dimensions of the velocity of money

Consider an economy in which the GDP for the year 2014 is $12,000 billion and in which the average quantity of money over the period is $1,000 billion. Then the average velocity of money for the year is 12.

Now, if we assume that production during the year is steady, the GDP for January 2014 is $1,000 billion. Since the average quantum of money during the year is assumed to be $1,000 billion we find that the velocity of money for January is just 1. The only way in which the velocity in January can be equal to the average velocity for the year is if the quantum of money in January were a twelfth of $1,000 billion. If the quantum of money for January is 1/12 times the average then the quantum of money in some other month will have to be 12 times the average. But we know that money does not grow and contract to the extent of 144 times in the space of a single year.

On the other hand, if we assume that money is more or less constant, then for the month of December 2014 (or any other month of the year) if we again assume that GDP was $1,000 billion we find that the velocity of money is 1 (or close to 1). For each month in the year we find that the velocity is 1 (or close to 1) whereas the velocity for the year as a whole is 12.

But this is clearly impossible.

Arithmetically, the reason is clear. In Case 1 to Case 4, changing the time period changes both the numerator and the denominator. But in the case of money velocity changing the time period changes only the numerator.

We are thus left with only one conclusion. The dimensions of the velocity of money are not t^{-1}. If the velocity of money for every sub-period is a fraction of the velocity for a whole period, it means that the velocity of money as we measure it is not the ratio of a flow to a stock but the ratio of two stocks: the stock of GDP for an arbitrary period (1 year in our case) to the average stock of money during that period. The velocity of money is a dimensionless number and has nothing to do with time.

Now what is the implication of this conclusion for economic theory. In the month of January 2014, the amount of money was equal to the GDP for that month. This holds equally for each of the 12 months. Once the $1000 billion is spent in January it cannot be spent again until the money returns in the form of income in the next month and so on. If all income flows were received twice a month $500 billion would be spent each 15 days and could not be spent again until it returned in the form of income flows in the next 15 days. The velocity would double but nothing else would change. The same total amount of money would be spent during the year and the GDP would be the same as before. If income flows were received once a year instead of once a month, 12 times the amount of money that is now paid out each month would have to be paid out at one go and this would be equal to $12,000 billion. If the velocity of money is 10, then this means that the amount of money received every 36.5 days is exactly equal to the amount of money spent during that period, ignoring saving.

Thus, at a fundamental level the velocity of money is 1; if the actual velocity that we measure is different it is merely because institutional reasons have decreed that income flows occur at intervals shorter than the interval over which GDP is conventionally measured.

At first glance we seem to have arrived at just another accounting identity whose physical meaning is the commonsense notion that you cannot spend money unless you have it.

But a closer look shows that the implications are far more revolutionary. All through the second half of the twentieth century, monetarists and Keynesians debated whether the Great Depression in particular and all recessions in general are caused by a contraction in money or a contraction in aggregate demand. If the velocity of money is just a dimensionless number, as we have conclusively shown, then this debate is pointless. A contraction in money is by definition a contraction in aggregate demand. At their core, monetarism and Keynesianism amount to the same thing.

Conclusion

The dispute between Keynesianism and monetarism has its basis in the misinterpretation of the dimensions of the velocity of money. Once it is recognised that the velocity of money is a dimensionless number between two stocks in two arbitrarily defined time periods, it turns out that a contraction in aggregate demand amounts to a contraction of the quantum of money, which in turns means that the basic Keynesian model and the basic monetarist model are the same. This does not imply that both of them are right but this is not the place to expand on that issue.

References

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