

# The dimensions of the velocity of money

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## **Introduction**

A mathematical equation that correctly describes a physical relationship between quantities is dimensionally homogeneous. [1] However, the converse is not true. An equation that is dimensionally homogeneous does not imply the existence of a physical relationship between the quantities in that equation. The dimensions of the velocity of money are generally taken to be  $t^{-1}$ . In what follows we examine this assumption and show that it is physically impossible. We then show what the correct dimensions of velocity are and arrive at the surprising inference that at their core Keynesianism and monetarism amount to the same thing.

## **The Quantity Theory**

The dimensions of the velocity of money are derived from the equation for the quantity theory of money.

$$M \cdot v = Y \tag{1}$$

where  $M$  is the quantity of money in an economy in dollars,  $v$  is its velocity and  $Y$  is the spending in a given time period. Since  $Y$  is usually taken to be the GDP and is measured in dollars per year, and  $M$  is measured in dollars,  $v$ , the residual, must be a number per year. Its dimensions are therefore  $t^{-1}$ .

The equation above has been the subject of considerable debate. But the debate is mainly about the stability of velocity and the causal relationship among the three variables in the equation. The equation itself is regarded as an accounting identity and therefore non-controversial. The dimensional homogeneity of the equation is taken to be proof of its physical truth. It is this assumption that we will now investigate.

We begin by examining variables which have a dimension  $t^{-1}$  and which figure in real world physical relationships.

## **Case 1**

The equation relating the frequency, wavelength and velocity of light is

$$f \cdot \lambda = c \tag{2}$$

where  $\lambda$  is the wavelength in metres,  $f$  is the frequency, and  $c$  is the speed of light in metres/second. The dimensions of frequency are thus  $t^{-1}$ , which are the same as the dimensions of the velocity of money.

If we conduct an experiment to test this relationship for red light over a period of 1 second, we find that light travels a distance of  $3 \times 10^8$  metres. Since the wavelength of red light is  $670 \times 10^{-9}$  m, over this distance we observe that the waveform repeats itself  $4.48 \times 10^{14}$  times. The frequency of light is therefore  $4.48 \times 10^{14}$ /sec.

If instead we conduct the experiment over a period of  $1/(3 \times 10^8)$  seconds, we find that light travels a distance of 1 m. Over this distance we would find that the waveform repeats itself 1492537.31343 times. Elementary arithmetic shows that the frequency of light is therefore  $4.48 \times 10^{14}$ /sec. In other words, the frequency of light is independent of the period of time over which we measure it.

## Case 2

A pipe carries water at a rate of  $25 \text{ cm}^3/\text{sec}$ .

We collect the water in a container over a period of 1 hour or 3,600 seconds and find that the volume of water collected is  $90,000 \text{ cm}^3$ . When the experiment is conducted over a period of 1 minute we find that the volume of water collected is  $1500 \text{ cm}^3$ .

In both cases we calculate and find that the flow rate is  $25 \text{ cm}^3/\text{sec}$ , which is of course what we would expect. As in Case 1, the result we get for the quantity whose dimensions are  $L^3t^{-1}$  is independent of the time period over which we choose to perform the measurement.

## Case 3

A stone is dropped from a height of 500 m. We assume that the acceleration due to gravity is  $10\text{m}/\text{sec}^2$ .

At the beginning of the fall the stone's velocity (dimensions:  $Lt^{-1}$ ) is 0 m/sec. At the end of 1 sec it is 10 m/sec. At the end of 9 seconds the velocity is 90 m/sec. And at the end of 10 seconds the velocity is 100 m/sec.

In the first second the stone travels a distance of 5m. The average velocity in the first second is thus 5m/sec. During the entire 10 seconds the stone travels a distance of 500 m. The average velocity over the entire period is thus 50 m/sec. In the tenth second the stone travels a distance of 95 m. The average velocity in the 10th second is thus 95 m/sec.

In this situation, where the quantity in which time figures as a  $t^{-1}$  dimension is increasing constantly, the ratio of the quantity in the first second to the quantity in the entire time period is less than 1. The ratio of the quantity in the last second to the quantity in the entire time period is greater than 1. And the ratio of the quantity in one time period to the

quantity in another time period (say, its value in the first second to that in the second second or its value in the second second to that in the third second) keeps varying.

#### **Case 4**

A stone thrown up from the ground at a speed of 100 m/sec is opposed by an acceleration of 10 m/sec<sup>2</sup>. It follows an exactly opposite trajectory as the stone in the previous example, slowing down until at a height of 500 metres its velocity becomes 0 m/sec.

In this case, the quantity where time figures as a  $t^{-1}$  dimension is slowing at a constant rate. The ratio of the quantity in the first time period to the quantity in the entire time period is greater than 1. The ratio of the quantity in the last time period to the quantity in the entire time period is less than 1. And the ratio of the quantity in one time period to the quantity in another time period keeps varying.

These results can be summed up as follows. When the quantity in which time figures as a  $t^{-1}$  dimension is constant, its measured value is independent of the time period over which we make the measurement. When the quantity in which time figures as a  $t^{-1}$  dimension is constantly increasing or constantly decreasing, then the ratio of its values in any two sub-periods is not constant. The ratio of its value in the first sub-period to its value in the total period of measurement is not equal to one.

#### **The dimensions of the velocity of money**

Consider an economy in which the GDP for the year 2014 is \$12,000 billion and in which the average quantity of money over the period is \$1,000 billion. Then the average velocity of money for the year is 12.

Now, if we assume that production during the year is steady, the GDP for January 2014 is \$1,000 billion. Since the average quantum of money during the year is assumed to be \$1,000 billion we find that the velocity of money for January is just 1. The only way in which the velocity in January can be equal to the average velocity for the year is if the quantum of money in January were a twelfth of \$1,000 billion, increasing 12 times to \$1,000 billion for the whole year, and we know that money does not grow at such a high rate.

Nor is that the only problem. For the month of December 2014 if we again assume that GDP was \$1,000 billion we find that the velocity of money is 1. For each month in the year we find that the velocity is 1 whereas the velocity for the year as a whole is 12.

But this is clearly impossible as we have observed in Cases 3 and 4 involving a  $t^{-1}$  quantity increasing or decreasing at a constant rate.

Arithmetically, the reason is clear. In Case 1 to Case 4, changing the time period changes both the numerator and the denominator. But in the case of money velocity changing the time period changes only the numerator.

We are thus left with only one conclusion. The dimensions of the velocity of money are not  $t^{-1}$ . If the velocity of money for every sub-period is a fraction of the velocity for a whole period, it means that the velocity of money as we measure it is not the ratio of a flow to a stock but the ratio of two stocks: the stock of GDP for an arbitrary period (1 year in our case) to the average stock of money during that period. The velocity of money is a dimensionless number.

There is nothing really surprising in this result. Assume that you receive a monthly income of \$1,000 in 2014. By running to the grocer instead of walking to the grocer you cannot increase the velocity of money. Similarly, if you spend all your money in the first 15 days of the month you cannot increase the velocity of money. If you receive \$12,000 during 2014 you cannot spend more than \$12,000. So long as the periodicity of income flows remains unchanged, nothing that you do in time can increase the velocity of money.

Now what is the implication of this conclusion for economic theory. In the month of January 2014, the amount of money was equal to the GDP for that month. This holds equally for each of the 12 months. Once the \$1000 billion is spent in January it cannot be spent again until the money returns in the form of income in the next month and so on. If all income flows were received twice a month \$500 billion would be spent each 15 days and could not be spent again until it returned in the form of income flows in the next 15 days. The velocity would double but nothing else would change. The same total amount of money would be spent during the year and the GDP would be the same as before. If income flows were received once a year instead of once a month, 12 times the amount of money that is now paid out each month would have to be paid out at one go and this would be equal to \$12,000 billion. If the velocity of money is 10, then this means that the amount of money received every 36.5 days is exactly equal to the amount of money spent during that period, ignoring saving. At a fundamental level the velocity of money is 1; if the actual velocity that we measure is different it is merely because institutional reasons have decreed that income flows are paid at intervals shorter than the interval over which GDP is measured.

At first glance we seem to have arrived at just another accounting identity whose physical meaning is the commonsense notion that you cannot spend money unless you have it.

But a closer look shows that the implications are far more revolutionary. All through the second half of the twentieth century, monetarists and Keynesians debated (and of course the debate continues) whether the Great Depression in particular and all recessions in general are caused by a contraction in money or a contraction in aggregate demand. If the velocity of money is just a dimensionless number, as we have conclusively shown, then this debate is pointless. A contraction in money is by definition a contraction in aggregate demand. At their core, monetarism and Keynesianism amount to the same thing.

## References

[1] Sonin, Ain. A, "The Physical Basis of Dimensional Analysis", Department of Mechanical Engineering, MIT, Cambridge, MA